SECTION A (40 MARKS)

Answer ALL questions in this section

1. Solve the equation $Sin3\theta = Cos\theta \ for \ 0^o \leq \theta \leq 180^o$. ***(05 marks)***

2. Given that $x^3 = (y - 3x)^2$ show that; $2x\frac{dy}{dx} = 3y - 3x$.
(05 marks)

3. $A(3,5)$ *and* $B(-5,-1)$ are points on a line. Find the coordinates of point C that divides $\overrightarrow{AB}$ in the ratio 3:1.
(a) Internally
(b) Externally
(05 marks)

4. Find the equation of the tangent to the curve $y = 1 + 2sinx \ at \ x = \frac{\pi}{4}$.
(05 marks)

5. Given that $z = 1 + 3i$ find the real numbers, $x \ and \ y$ such that $xZ + y\bar{Z} = 7 + 3i$. ***(05 marks)***

6. $O(0,0)$ *and* $Q(4,0)$ are fixed points. $P(x,y)$ is a variable point. Given that $< OPQ = 45^o$, find the locus of $P(x,y)$. ***(05 marks)***

7. When a polynomial $P(x)$ is divided by $x^2 - 4$, the remainder is $3x + 7$. find the remainder when $P(x)$ is divided by;
(a) $x - 2$
(b) $x + 2$
(05 marks)

8. Evaluate; $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3 - x^2}}$ without using tables.
(05 marks)

SECTION B (60 MARKS)

9. (a) Express $\sqrt{3}\, Sin\theta + Cos\theta$ in the form $Rsin(\theta + \alpha)$. ***(03 marks)***
Hence solve the equation $\sqrt{3}Sin\theta + Cos\theta = \sqrt{2}$,
for $0^o \leq \theta \leq 360^o$. ***(03 marks)***

(b) Prove the Cosine rule $a^2 = b^2 + c^2 - 2bc\, CosA$ for any triangle ABC. Hence solve the triangle in which $b = 5cm$, $c = 8cm$ *and* $A = 60^o$. ***(06 marks)***

10. (a) Use the mathematics of small changes to evaluate $Sin\ 29.5^o$ to 4 dpls. ***(05 marks)***

(b) A right circular cone has a slant length of $9\sqrt{3}cm$. Calculate the maximum volume of the cone; and state the corresponding values of the height and the radius in this case. ***(07 marks)***

11. (a) Find the term in x^{-3} in the expansion of;
$\left(x^2 + \frac{1}{2x}\right)^9$ ***(04 marks)***

(b) Expand $\sqrt{1 - \frac{1}{4}x}$ up to the term in x^3; and use the expansion to evaluate;
(i) $\sqrt{15}$ *to 3dps*
(ii) $\sqrt{7}$ *to 4dps* ***(08 marks)***

12. Express $\frac{x^6 + 64}{x^4 - 16}$ in partial fractions; hence evaluate;

$$\int_3^4 \frac{x^6 + 64}{x^4 - 16}\, dx \quad to\ 4dps$$

(12 marks)

13. ✓(a) Find the coordinates of the point of intersection of the lines;

$$\boldsymbol{r_1} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \text{ and } \boldsymbol{r_2} = \begin{pmatrix} 10 \\ 1 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix};$$

and compute the acute angle between the lines. *(09 marks)*

(b) Write down the vector equation of the plane containing the two lines in (a) above. *(03 marks)*

14. Given the curve $y = \frac{2x - 5}{x^2 - 4}$,

(a) Find its stationary points; hence state the region within which the curve does not lie. *(06 marks)*

(b) Sketch the curve, and deduce the solution to the inequality $\frac{2x - 5}{x^2 - 4} \geq 0$ *(06 marks)*

15. (a) Prove that the equations of the tangent to the parabola $y^2 = 4ax$ At the variable point $(at^2, 2at)$ *is* $x - ty + at^2 = 0$. *(03 marks)*

(b) Deduce the equations of the tangents to the parabola $y^2 = 4ax$ from the external point $A(-6a, a)$; *(04 marks)*

Hence (i) find the coordinates of the points of contact of the tangents with the parabola. *(02 marks)*

(ii) show that the tangents make 45° with each other. *(03 marks)*

16. ✓ The temperature, $\theta^o C$, at a height ***h*** metres risen above the foot of a 1000 m high mountain, decreases at a rate which is directly proportional to the height risen.

(a) A tourist notices that the temperature of water drops from 16°C, at the foot of the mountain, to -9°C at the peak of the mountain. Set up a differential equation for this problem and solve it. *(06 marks)*

(b) Calculate the;

(i) height at which water starts to freeze. *(03 marks)*

(ii) temperature the water should have at the foot of the mountain if it just freezes at the top of the mountain. *(03 marks)*

END